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ON-LINE IDENTIFICATION AND ESTIMATION OF NON-LINEAR STIFFNESS PARAMETERS OF BEARINGS

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1. EQUATION OF MOTION AND RESPONSE

The governing equation for a balanced rigid rotor supported at ends in bearings with non-linear stiffnesses is written as [1]

$$\ddot{x} + 2\xi\omega_n\dot{x} + G(x) = \zeta(t), \tag{1}$$

where $G(x)$ is an unknown function. The random excitation to the system is represented by $\zeta(t)$. The approach to the solution of equation (1) is greatly simplified if the overall random excitation to the system, from the variety of sources, is treated as ideal white noise. The steady state solution of equation (1), in terms of joint probability density function, can be obtained as

$$p(x, \dot{x}) = c\exp[-(2\xi\omega_n/\pi S_0)\{\tfrac{1}{2}\dot{x}^2 + g(x)\}], \tag{2}$$

where S_0 is the uniform spectral density of excitation, c is a normalization constant and

$$g(x) = \int_0^s G(\eta)\,\mathrm{d}\eta.$$

The probability density functions $p(x)$ and $p(\dot{x})$ are obtained from equation (2) and the variance of the velocity response is obtained from $p(\dot{x})$. Using the velocity variance the probability density function for the displacement response can be written as [1]

$$p(x) = c_1\exp[-(1/\sigma_{\dot{x}}^2)g(x)] \tag{3}$$

with

$$c_1 = 1\Big/\int_{-\infty}^{\infty}\exp[-(1/\sigma_{\dot{x}}^2)g(x)]\,\mathrm{d}x.$$

2. IDENTIFICATION AND ESTIMATION

The objective of the identification procedure is to detect the form of the non-linear function $G(x)$. On taking logarithm on both sides and differentiating with respect to x, equation (3) gives

$$(1/p(x))\{dp(x)/dx\} = (-1/\sigma_{\dot{x}}^2)G(x). \quad (4)$$

Equation (4) can be rewritten in more convenient form as

$$G(x) = (-\sigma_{\dot{x}}^2)\{1/p(x)\}\{dp(x)/dx\}. \quad (5)$$

The above equation is a representation of the restoring force function $G(x)$ in terms of the displacement and velocity response of the rotor-bearing system. Variance, $\sigma_{\dot{x}}^2$, probability function, $p(x)$, and its derivative, $dp(x)/dx$, can be computed from the experimentally measured displacement and velocity data (x and $\dot{x}$), which enables the reconstruction of function $G(x)$.

3. ILLUSTRATION

The procedure is illustrated on a laboratory rig consisting of a disc centrally mounted on a shaft supported in two identical ball bearings. The shaft is driven through a flexible coupling by a motor and the vibration signals are picked up (after balancing the rotor) in both, the vertical and horizontal directions, by accelerometers mounted on one of the bearing housings. The signals from the accelerometers are digitized on a PC/AT after magnification.

Typical displacement and velocity signals in the vertical direction are picked up by the accelerometer the non-linear restoring force, $G(x)$ of the bearing estimated from equation (5) is shown in Figure 1. The trend indicates the presence of a softening type of non-linearity. The form can be approximated to the desired degree of accuracy through a polynomial in x. Employing the simplest representation, $G(x)$ can be expressed as

$$G(x) = \omega_n^2(x - \lambda x^3). \quad (6)$$

The bearing parameters, ω_n^2 and λ can now be readily obtained through equation (6) and Figure 1. Bearing parameters thus obtained for the displacement and velocity signals and two other such sets are given in Table 1.

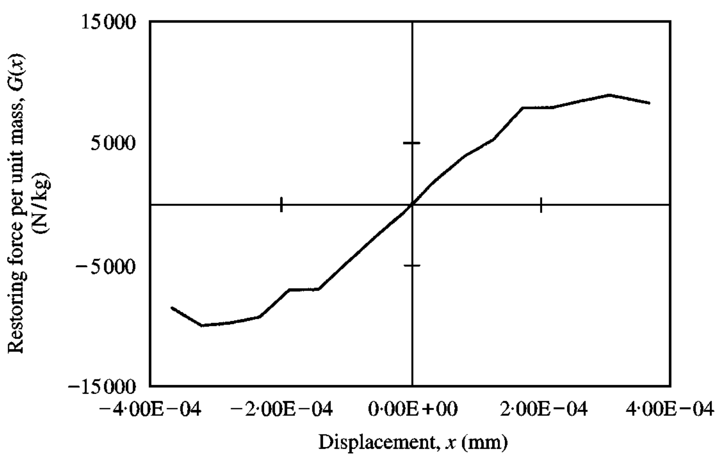

Figure 1. The variation of the non-linear restoring force.

TABLE 1

Estimated parameters (vertical direction)

Signal set	ω_n^2 (rad/s)^2	λ (mm^{-2})	$G(x)$ (N/kg)
1	5·42E7	−1·27E6	5·42E4x − 6·88E10x^3
2	5·37E7	−1·24E6	5·37E4x − 6·66E10x^3
3	5·32E7	−1·23E6	5·32E4x − 6·54E10x^3

TABLE 2

Estimated parameters (horizontal direction)

Signal set	ω_n^2 (rad/s)^2	λ (mm^{-2})	$G(x)$ (N/kg)
1	3·21E7	−1·29E6	3·21E4x − 4·14E10x^3
2	3·14E7	−1·24E6	3·14E4x − 3·89E10x^3
3	3·11E7	−1·25E6	3·11E4x − 3·89E10x^3

A similar exercise with displacement and velocity signals in the horizontal direction yields the parameter values listed in Table 2.

4. VALIDATION

The values of the bearing stiffness parameters ω_n^2 and λ, obtained by the procedure outlined, are validated by comparison with those obtained from the analytical formulations of Harris [2] and Ragulskis *et al.* [3], which are based on Hertzian contact theory.

It can be seen that the bearing stiffness is critically dependent on the preloading, g, of the balls. While the manufacturer may, at times, provide the preload range, the exact value of the preloading of the bearing balls in the shaft-casing assembly, especially during operations which have involved wear and tear, would be difficult to determine. It is also to be noted that the theoretical stiffness calculations are based on formulations which analyze the bearing in isolation of the shaft. The theoretically possible stiffnesses are listed in Table 3. The expressions for the theoretical stiffnesses in Table 3 have been obtained by curve fitting the stiffness values obtained from the analytical formulations of Harris [2] and Ragulskis *et al.* [3], through a quadratic in x.

The stiffness parameter estimates of Tables 1 and 2, from the procedure developed, show good agreement with the results of Table 3, obtained through available analytical formulations for isolated ball bearings. The advantage of the proposed methods over other available techniques is distinct from the fact that it does not involve measurement of the excitation forces and works directly on the random response signals, which can be conveniently picked up at the bearing caps and also that the procedure does not require the damping in the system to be known.

5. SIMULATION CHECK FOR MEASUREMENT ERRORS

A computer simulation is carried out to obtain an estimate for the robustness of the procedure, in the presence of measurement noise. The actual noise-to-signal ratio in the

TABLE 3

Theoretical bearing stiffness parameters [2, 3]

Preload (mm)	$G(x)$ (N/mm)
0·0002	$1{\cdot}20\times10^4 - 4{\cdot}01\times10^{10}x^2$
0·0003	$1{\cdot}47\times10^4 - 2{\cdot}18\times10^{10}x^2$
0·0004	$1{\cdot}69\times10^4 - 1{\cdot}42\times10^{10}x^2$
0·0005	$1{\cdot}89\times10^4 - 1{\cdot}02\times10^{10}x^2$
0·0006	$2{\cdot}08\times10^4 - 6{\cdot}09\times10^{9}x^2$

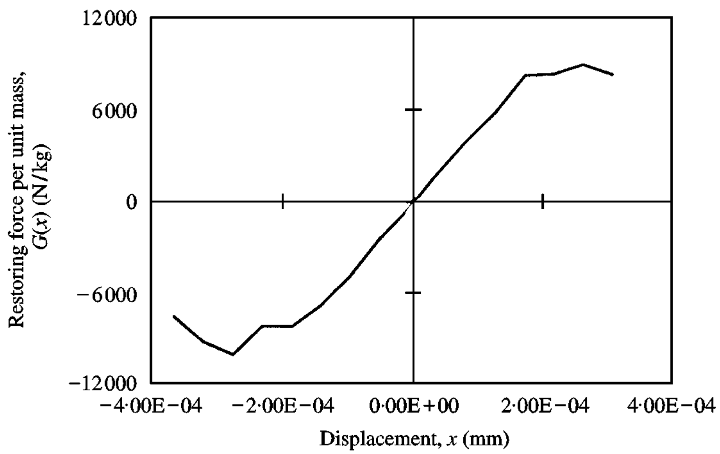

Figure 2. The variation of the non-linear restoring force, without measurement noise.

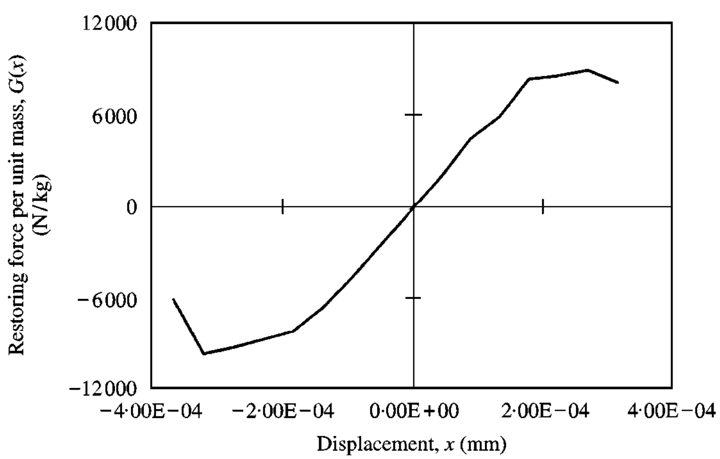

Figure 3. The variation of the non-linear restoring force, with 1% measurement noise.

instruments used for measurement of displacement and velocity signals was found to be less than 1%.

For the purpose of computer simulation two cases are considered—(i) without noise and (ii) 1% random noise in the measurement of both—displacement and velocity. The displacement and velocity signals are simulated, through Monte–Carlo simulation of a non-linear system with restoring force being given by a simplest type of restoring force, $G(x)$, $G(x) = \omega_n^2 x - \lambda\omega_n^2 x^3$ ($\omega_n^2 = 5{\cdot}42 \times 10^7\,(\text{rad/s})^2$ and $\lambda = -1{\cdot}27 \times 10^6$ (mm^{-2})). The restoring force, $G(x)$, is estimated from equation (5) and is shown in Figure 2. For the second case, 1% random noise is added to the measured signals. Figure 3 is the estimated restoring force $G(x)$ for the noisy displacement signal.

The values for $G(x)$, obtained for the two cases are

Without noise $(5{\cdot}42 \times 10^4 x - 6{\cdot}60 \times 10^{10} x^3)$,

With 1% noise $(5{\cdot}23 \times 10^4 x - 6{\cdot}81 \times 10^{10} x^3)$.

The closeness of the above values suggests that the effect of measurement noise on the estimates is minimal. A more rigorous error analysis can be carried out with non-dimensional parameters and for both hardening and softening restoring force to firmly establish the robustness of the algorithm.

CONCLUSION

The procedure has a distinct advantage over existing ones in that it does not require a known input force for excitation of the system. It works directly on the naturally available response signals of the machine. The procedure carries out identification of the form of the non-linearity and also provides estimates of the parameters. The results show good agreement with the theoretical possible values for isolated bearings. The computer simulation exhibits a check for the degree of error due to possible measurement noise.

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